HL Number and Operations Practice Questions
03/01/2007

Student Name: ______________________________

Class: ______________________________

Date: ______________________________

Instructions: **Read each question carefully and select the correct answer.**

1. There are nine players on the softball team. Which of the following formulas would you use to determine how many different ways there are to arrange the batting order?

A. 9 x 9 = n
B. 9 x 1 = n
C. 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1= n
D. 9 + 8 + 7 + 6 + 5 + 4 +3 + 2 + 1 = n

2. How many ways can five friends sit at a dinner table (in five seats)?

A. 5
B. 120
C. 20
D. 25

3. Find the value of n in the table.

Number of chairs in a row	Number of people to sit in the chairs	Number of arrangements
4	9	n

A. 3,024
B. 36
C. 1,296
D. 216

4. There are ten people interviewing for two computer programmer positions. How many different ways can the company choose a programmer and a junior programmer?

A. 2!
B. 18!
C. 80
D. 90

5. There are twelve students trying out for the varsity tennis team. Only nine students will be chosen. How many different combinations can the coach choose for the team?

A. 6
B. 108
C. 220
D. 1320

6. There are 12 teachers. Three are selected to lead the field trip. How many different groups of three teachers can be formed?

A. 22
B. 440
C. 220
D. 44

7. At a restaurant, customers can select 3 toppings for their salad. Each topping must be different. The customers can select from mushrooms, carrots, sprouts, cucumber, bacon bits, and croutons. How many different ways could a customer top their salad?

A. 2
B. 18
C. 20
D. 720

8. Carlos works at a frozen yogurt store. Customers can select two different toppings for their yogurt. Customers can select fruit, cookie crumbs, or chocolate bits. How many ways could a customer top a yogurt?

A. 2
B. 3
C. 6
D. 5

9. A family of 5 (2 adults and 3 children) are at a masquerade party. The children are wearing masks, so you cannot discern the sex of the 2 older children. The youngest child is a boy. What is the probability that this family has at least 1 other boy?

A. 1/4
B. 1/2
C. 2/3
D. 3/4

10. Janet tossed one nickel. Then, she tossed another nickel. What is the sample space for both tosses?

A. HHH, TTT, HTH, THT
B. HH, HT, TT,
C. HH, HT, TH, TT
D. HHH, TTT, HTH

11. You roll one die and then toss one coin. Find the probability that the number on the die is 4 or less, and that the coin is heads.

A. 1/8
B. 1/2
C. 1/3
D. 1/4

12. The human resources department can estimate the number of positions that will experience turnover in a calendar year based on the economy, the company's success, and the working conditions. All of these factors show that this year has a 78% chance of being a good year. In a good year, 8 positions experience turn over. In a bad year, 27 positions experience turn over. According to this information, how many positions are expected to turn over this year?

A. 12 positions
B. 6 positions
C. 21 positions
D. 22 positions

HL Number and Operations Practice
Answer Key
03/01/2007

1. There are nine players on the softball team. Which of the following formulas would you use to determine how many different ways there are to arrange the batting order?

C. 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1= n
Permutations

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Permutations

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Permutations

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D. 90
Permutations

5. There are twelve students trying out for the varsity tennis team. Only nine students will be chosen. How many different combinations can the coach choose for the team?

C. 220
Combinations

6. There are 12 teachers. Three are selected to lead the field trip. How many different groups of three teachers can be formed?

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7. At a restaurant, customers can select 3 toppings for their salad. Each topping must be different. The customers can select from mushrooms, carrots, sprouts, cucumber, bacon bits, and croutons. How many different ways could a customer top their salad?

C. 20
Combinations

8. Carlos works at a frozen yogurt store. Customers can select two different toppings for their yogurt. Customers can select fruit, cookie crumbs, or chocolate bits. How many ways could a customer top a yogurt?

B. 3
Combinations

9. A family of 5 (2 adults and 3 children) are at a masquerade party. The children are wearing masks, so you cannot discern the sex of the 2 older children. The youngest child is a boy. What is the probability that this family has at least 1 other boy?

D. 3/4
Probability

10. Janet tossed one nickel. Then, she tossed another nickel. What is the sample space for both tosses?

C. HH, HT, TH, TT
Probability

11. You roll one die and then toss one coin. Find the probability that the number on the die is 4 or less, and that the coin is heads.

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12. The human resources department can estimate the number of positions that will experience turnover in a calendar year based on the economy, the company's success, and the working conditions. All of these factors show that this year has a 78% chance of being a good year. In a good year, 8 positions experience turn over. In a bad year, 27 positions experience turn over. According to this information, how many positions are expected to turn over this year?

A. 12 positions
Probability

Study Guide

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03/01/2007

Permutations

Permutations refer to the different ways that the items of a group can be arranged. For example: If 4 people are sitting in a row of 4 chairs, how many permutations (ways of arranging the group) are there? There are 4 people who could sit in the first chair, 3 left to sit in the second chair, 2 who could sit in the third chair, and only 1 left to sit in the fourth chair. From this we can use the factorial (4! = 4 x 3 x 2 x 1) to find the answer. There are 24 ways to arrange them since 4 x 3 x 2 x 1 = 24.

Example 1: If there are 3 students, how many ways can these 3 students be arranged in 3 chairs?

Solution: Any one of the three students can fill the first chair. Once the first chair is filled, only two students are left to fill the second chair. Only 1 student will be left to fill the final chair. This relationship can be represented by a factorial as follows.

3! = 3 x 2 x 1 = 6

Answer: There are 6 possible arrangements (permutations) for the students to fill the chairs.

Permutations are usually written in the form ${}_xP_y$ P stands for Permutation
x stands for the number of people/objects, etc.
y stands for the number of positions to fill

Example 2: The are 8 contestants in the drawing contest. How many different ways could 1st through 4th place be chosen?

Solution: We must determine the number of possibilities for the first 4 places. There are 8 possibilities for 1st place, 7 possibilities for 2nd place (once the first place winner is chosen), 6 possibilities for 3rd place, and 5 possibilities left for 4th place. We stop here because there are only 4 places to fill. We write this as:

$${}_8P_4 = 8 \times 7 \times 6 \times 5$$
$${}_8P_4 = 1{,}680$$

The answer is 1,680.

Combinations

A combination is a selection of items in which the order does not matter. In combinations, the number of items to choose from exceeds the number of slots for those choices. The formula to compute the number of combinations requires student to use factorials. (To understand combinations, students must understand permutations and factorials.) Combinations are used to understand the number of different ways an event can occur.

Students must first understand the formula used for solving problems with combinations. In the following equation for determining the number of combinations, n stands for the number of choices and r stands for the number of available slots or positions.

$$\frac{n!}{r!(n-r)!}$$

Example: Jim can bring 3 friends to the park and he has 6 friends to choose from. How many different combinations of friends can Jim take to the park?

(1)

$n = \text{number of choices}$

$n = 6$

(2)

$r = \text{available slots}$

$r = 3$

(3)

$$\frac{n!}{r!(n-r)!}$$

(4)

$$\frac{6!}{3!(6-3)!}$$

(5)

$$\frac{6!}{3!3!}$$

(6)

$$\frac{6\times5\times4\times3\times2\times1}{(3\times2\times1)(3\times2\times1)}$$

(7)

$$\frac{720}{36} = 20$$

<u>Step 1</u>: Identify the value of n.
<u>Step 2</u>: Identify the value of r.
<u>Step 3</u>: Select the formula for determining the number of combinations.
<u>Step 4</u>: Substitute the values of n and r into the formula for determining the number of combinations.
<u>Step 5</u>: Simplify the expression by subtracting: 6 - 3 = 3.
<u>Step 6</u>: 6! = 6 x 5 x 4 x 3 x 2 x 1and 3! = 3 x 2 x 1. Replace 6! and 3! with the expanded equivalents.
<u>Step 7</u>: Multiply to simplify the expression. 6 x 5 x 4 x 3 x 2 x 1 = 720 and (3 x 2 x 1)(3 x 2 x 1) = 36
Divide 720 by 36 to get the number of combinations.

Answer: There are 20 different combinations of friends.

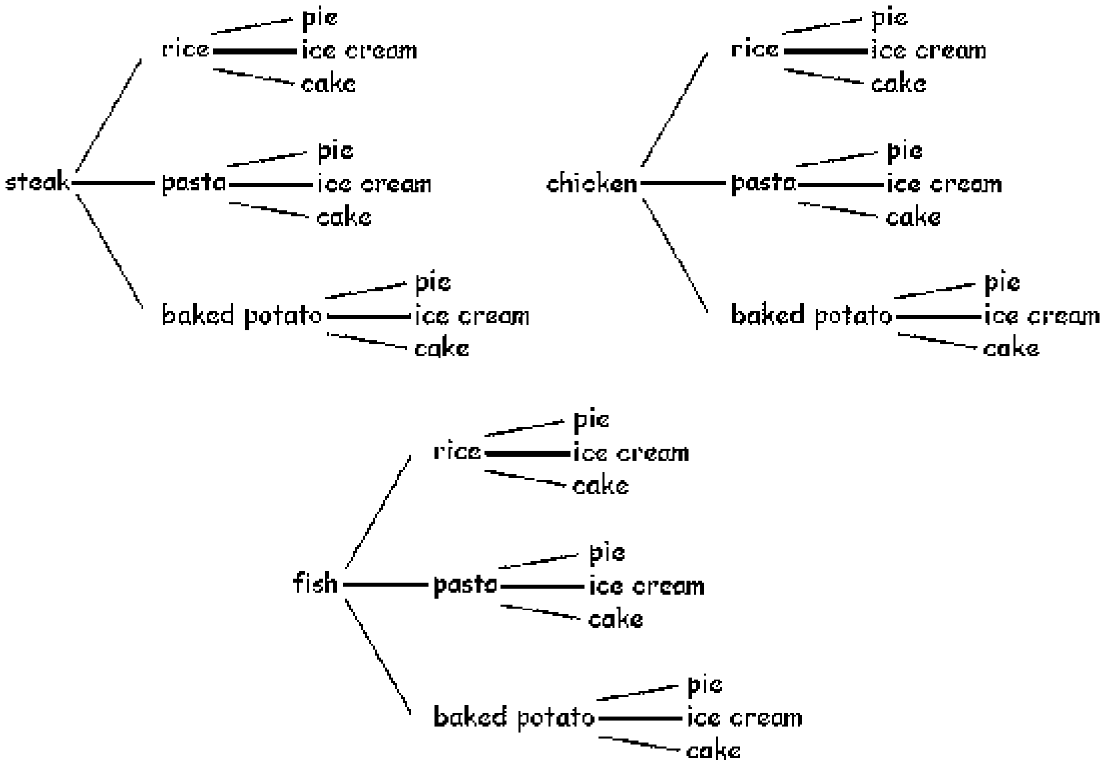
Probability

<u>Probability</u> is the measure of the chance that a specific outcome will occur. Probability methods at this level include using tree diagrams, sample space, the fundamental counting principle, adding and multiplying probabilities for independent and dependent events, calculating expected value, conditional probability, experimental probability, and theoretical probability.

<u>Tree diagrams</u> are probability tools which represent possible outcomes. If you went to dinner at a banquet, you may be presented with the following possibilities:

Main dishes: steak, fish, chicken
Side dishes: rice, pasta, baked potato
Dessert: pie, ice cream, cake

Suppose that at this dinner you were asked to choose one item from each category: one main dish, one side dish, and one dessert. How many different possible meals could you choose? A tree diagram which gives the sample space (the choices) would help you quickly count the choices:



The above illustration is a tree diagram. All that is left to do is to count the choices down the right side of each branch: there are 27 different possible meals.

Two events are <u>independent</u> if the probability of one event happening has no influence on the probability of the other event happening. If you roll one die and toss one coin, you know that the number on the die has nothing to do with whether the coin toss results in heads or tails. The formula for determining the probability that two independent events will occur is below.

P(A and B) = Probability of A x Probability of B = P(A) x P(B)

Example 1: What is the probability a coin toss resulting in heads and a roll of the die resulting in a 3 or less?

(1) P(A and B) = P(A) x P(B)
(2) P(A and B) = 1/2 x 3/6
(3) P(A and B) = 1/2 x 1/2
(4) P(A and B) = 1/4

<u>Step 1</u>: Choose the correct formula for the probability of A and B happening.
<u>Step 2</u>: There are only two possibilities when tossing a coin (heads and tails), so the probability of a coin toss resulting in heads is 1/2. There are six possibilities when rolling a die (1, 2, 3, 4, 5, and 6). Only three of those possibilities are equal to or less than 3, so the probability of the roll of a die resulting in a 3 or less is 3/6. Substitute the probabilities into the formula.
<u>Step 3</u>: Reduce the fractions before multiplying.
<u>Step 4</u>: 1/2 times 1/2 equals 1/4. Remember to multiply numerators and denominators straight across.

Answer: 1/4

<u>Dependent events</u> are events which influence one another's probability of occurring. The formula for determining the probability that two dependent events will occur is below.

P(A and B) = Probability of A x Probability of B given A = P(A) x P(B, given A)

Example 2: If you draw one card from a deck, put it aside, and then draw another card, what is the probability that each card drawn is a heart?

(1) P(A and B) = P(A) x P(B, given A)
(2) P(A and B) = 13/52 x 12/51
(3) P(A and B) = 156/2652
(4) P(A and B) = 13/221

Step 1: Choose the correct formula for the probability of A and B happening.
Step 2: There are 52 cards in a deck of cards. 13 of the cards in each deck are hearts. The probability that the first card drawn is a heart is 13/52. The probability that the second card drawn is a heart is 12/51 because there is one less heart in the deck and one less card in the deck. Substitute the probabilities into the formula.
Step 3: Multiply the fractions. Remember to multiply numerators straight across and denominators straight across.
Step 4: Reduce the fraction completely.

Answer: 13/221

The formula for calculating expected value is:
(E = result of outcome #1 x probability of a outcome #1 + result of outcome #2 x probability of outcome #2).
Businesses can use such a formula to roughly project expected profits under specific conditions.

Example 3: Suppose you owned a snack bar at a beach. Let's say that in a good summer you make $3,000 and in a bad summer you lose $50. The greatest determining factor of a good or bad year has been the weather, and all indications show that the approaching summer season has an 89% chance of being sunny and warm - a good year. What is your projected profit for the approaching season?

(1) E = ($3,000 x 0.89) + (-$50 x 0.11)
(2) E = ($2,670) + (-$5.50)
(3) E = $2,664.50

Step 1: The result of a good summer is $3,000 and the probability that there will be a good summer is 89% (0.89). The result of a bad summer is losing $50 (-$50) and the probability that there will be a bad summer is 11% (0.11). Use these values to fill in the formula for calculating the expected value.
Step 2: Multiply $3,000 by 0.89 to get $2,670 and multiply -$50 by 0.11 to get -$5.50.
Step 3: Add the results of Step 2.

The expected profit for the approaching summer season is $2664.50.

To calculate conditional probability, you must find the probability of an event based on the fact that another event has already happened.

Example 4: An algebra class gets a new student, a girl. This new student happens to have two younger siblings. Find the probability that one of the new student's siblings is also a girl.

Solution: Examine all of the possible ways three siblings might be arranged in terms of their gender. The fact that the first sibling, the new girl in class, is a girl alters the possible choices for the problem. The possibilities for the genders of 3 siblings are: GGG (Girl, Girl, Girl), GGB, GBG, GBB, BGG, BGB, BBG, BBB. From these possibilities, you can cancel out any that don't begin with G since we know that the oldest sibling is a girl. That leaves us with 4 possibilities: GGG, GGB, GBG, and GBB. Three of these result in two siblings that are girls. Therefore the probability that at least two of the siblings are girls is 3/4 or 75%.

Experimental probability is a way to predict future events using data from past events. Experimental probability is calculated by dividing the number of occurrences of an event by the number of trials of an experiment. A football coach, for example, can predict how well his receiver will complete passes. If the receiver has been completing 10 out of every 25 passes thrown to him, then the coach can use experimental probability to predict how well he will complete passes in the next game: 10/25 = 0.4 or 40%. The prediction is that the receiver will complete 4 out of 10 or 2 out of 5 passes thrown to him.

In contrast, theoretical probability or mathematical probability refers to finding the probability of an event before any trials of an experiment have been performed. Often theoretical or mathematical probability is referred to as just probability.

If you want to find the probability of rolling a die and getting a 4, you simply set up the fraction 1/6 (1 because there is only one 4 on the die and 6 because there are six sides on the die meaning six different possible outcomes.) Therefore, before we even roll a die, we know that theoretical probability tells us that we have a 1 in 6 chance of rolling a 4.

The Counting Principle

Counting the choices involves determining how many choices are available in a given situation. If there are A choices for one way and B choices for another, then the total number of choices is A x B.

Example 5: Ana went to the world's largest amusement park. There were 10 different rides, 14 roller coasters, 8 shows, and 6 shops. How many different ways can Ana see all of the attractions?

Solution: Multiply 10 x 14 x 8 x 6 = 6,720

Answer: 6,720 ways